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INTEGRATION OF IRRATIONAL FUNCTION

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$$\int \frac{dx}{\sqrt{x^2+a^2}}$$

Put $x = a \sinh \theta \quad \therefore dx = a \cosh \theta$

$$\therefore I = \int \frac{a \cosh \theta d\theta}{\sqrt{a^2 \sinh^2 \theta + a^2}} = \int \frac{a \cosh \theta d\theta}{a \cosh \theta}$$

$$\Rightarrow \int d\theta$$

$$\Rightarrow \theta = \sinh^{-1} \frac{x}{a}$$

$$= \log \left(\frac{x + \sqrt{x^2 + a^2}}{a} \right) + C$$

$$= \log(x + \sqrt{x^2 + a^2}) - \underbrace{\log a}_{C} + C$$

$$= \log(x + \sqrt{x^2 + a^2})$$

$$\textcircled{v} \int \frac{dx}{\sqrt{x^2 - a^2}} \quad x > a$$

Put $x = a \cosh \theta \quad \therefore dx = a \sinh \theta d\theta$

$$\therefore I = \int \frac{a \sinh \theta d\theta}{\sqrt{a^2 \cosh^2 \theta - a^2}} =$$

$$\Rightarrow \int \frac{a \sinh \theta}{a \sqrt{\cosh^2 \theta - 1}} d\theta$$

$$\Rightarrow \int \frac{a \sinh \theta}{a \sinh \theta} d\theta$$

$$\Rightarrow \int d\theta = \theta = \cosh^{-1} \frac{x}{a}$$

$$= \frac{\log x + \sqrt{x^2 - a^2}}{a}$$